MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM ID 2220)

EME4076 – MECHANICAL VIBRATIONS

(ME)

11 July 2023
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 11 pages with 4 Questions, excluding the cover page.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 Marks)

Part 1

Figure Q1_1(a) shows a floor tile of 0.2 m by 0.2 m at an industrial location. The tile is made of a material with Young's Modulus of 3.9 GPa. Figure Q1_1(b) shows the simplified model of the floor tile when it is subjected to vibration; it is modeled as a load of $m = 0.072$ kg that is mounted on a very short vertical column with an original length of $l_0 = 0.0015$ m, which is the thickness of the tile. Figure Q1_1(c) shows the free-body diagram of the mass at time t, together with its kinematics when displacement x is directed upwards.

x is the change in length of the column and $\ddot{x}$ is the acceleration of the change in length.

The initial compression of the column is 2×10^{-10} m, just before the compression is released and the vibration begins. The initial velocity is zero.

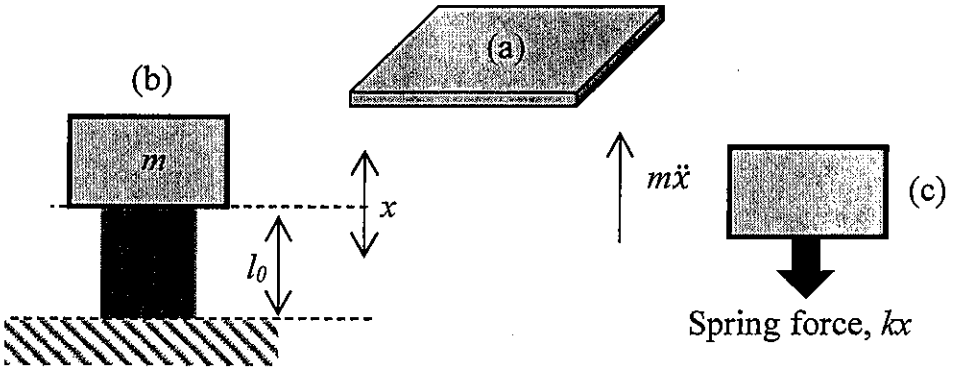

Figure Q1_1

a) For this model, the effect of the weight of the load m on the column is not considered in Figure (c), despite the vertical orientation. Briefly infer the reason for this. (*Hint: the effect of the load is not negligible.*)

[2 marks]

b) The column must be modelled as a spring. Find the equivalent stiffness of the column. (*Hint: the cross section of the column is equal to the surface of the tile.*)

[2 marks]

c) The equivalent stiffness of the column is expected to be practically constant, despite the changes in length. Briefly infer the reason for this.

[2 marks]

d) Provide a brief example of what could have caused the initial compression, assuming an industrial location.

[2 marks]

Continued...

e) Determine the natural frequency ω_n of the model.

[2 marks]

f) Apply $\Sigma F = m\ddot{x}$ and present the answer as a homogeneous differential equation.

[2 marks]

g) Determine the displacement response equation x for the model.

[2 marks]

Part 2

Referring to Part 1, the model would continue vibrating due to the lack of any damping; this is not desirable for a floor tile in an industrial location. Therefore, damping is to be introduced into the model as a layer that is vertically parallel to the column, as shown in Figure Q1_2(a). The damping layer can be modelled as a damper that is parallel to the equivalent spring of the column. Figure Q1_2(b) shows the free-body diagram.

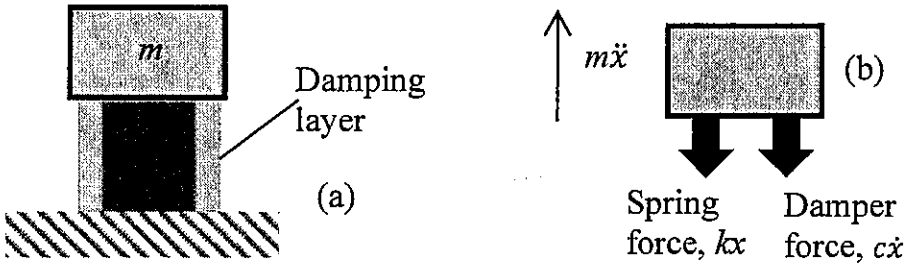

Figure Q1_2

The damping must be critical.

a) Briefly propose a method of implementing the damping layer in a real and practical situation. *(Note: Observe Figure 1_1(a), i.e. the floor tile)*

[2 marks]

b) Figures 1_1(c) and 1_2(b) show that the spring and damper forces are opposite the direction of x. Briefly describe the reason for this.

[1 mark]

c) The damping could have been other than critical. Briefly explain a reason for having the damping be critical and how this is necessary for the purpose of the floor tile.

[2 marks]

d) Determine the critical damping coefficient.

[1 mark]

e) Apply $\Sigma F = m\ddot{x}$ and present the answer as a homogeneous differential equation.

[2 marks]

h) Determine the displacement response equation x for the model.

[3 marks]

Continued...

Question 2 (25 Marks)

Figure Q2(a) shows a horizontal plunger in a machine. Figure (b) shows the schematic representation of Figure (a) as a horizontal mass-spring system in one-degree translation. Coulomb damping is significant in this system.

The initial displacement x_0 is due to a pressure difference between the left and right sides of the plunger. x_0 is positive, i.e. to the right The initial velocity $\dot{x}_0$ is zero.

The plunger's equivalent stiffness is $k = 200$ N/m.
The friction coefficient of the walls is $\mu = 100/981$.
The mass of the plunger is $m = 0.05$ kg.
The initial displacement is $x_0 = 0.001\ m$

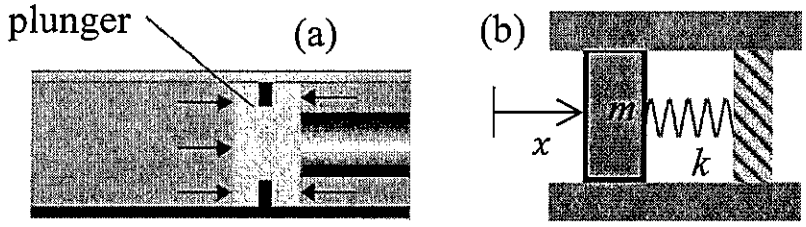

Figure Q2

a) Briefly infer how the vibration can begin, given the initial displacement.
[2 marks]

b) Determine the first phase of the **first cycle**: $\dot{x} > 0$, or $\dot{x} < 0$.
[1 mark]

c) Determine the value for the term $\frac{\mu mg}{k}$.
[1 mark]

d) Determine the natural frequency ω_n.
[2 marks]

e) Apply the initial condition of x_0.
[2 marks]

f) Apply the initial condition of $\dot{x}_0$.
[2 marks]

g) Determine the response equations x and $\dot{x}$ for the *first phase* of the **first cycle**.
[1 mark]

h) Determine the time t_1 when the plunger changes direction for the first time.
(*Hint/Reminder: A change in direction generally happens when velocity reaches zero.*)
[3 marks]

Continued...

i) Determine the displacement x_1 at time t_1.
(This is also the starting displacement for the *second phase* of the **first cycle.**)
[1 mark]

j) Determine the response equations x and $\dot{x}$ for the *second phase* of the **first cycle.** (*Use the answer for part i) as the starting condition for this equation.*)
[4 marks]

k) Determine the time t_2 when the plunger changes direction for the second time.
(*Note: Keep in mind that* $t_2 > t_1$.)
[2 marks]

l) Determine the starting displacement x_2 for the **second cycle.**
[2 marks]

m) From the answers to parts k) and l), determine if there is any possibility for a second cycle.
[2 marks]

Continued...

Question 3 (25 Marks)

Part 1

Figure 3_1 shows a tachometer check on a motor. The motor is briefly started and then the power supply to the motor is gradually reduced to determine its response without the application of brakes.

Figure 3_1

This scenario can be modelled as an **underdamped** system with an excitation; it is not in resonance. The equation of motion for this system is extrapolated as follows:

$$10\ddot{x} + 20\dot{x} + 40x = 3e^{-t}$$

The initial conditions are zero.

a) From the equation of motion, determine the following (in their appropriate units):
   (i) Stiffness k
   (ii) Mass of the system m
   (iii) Natural frequency ω_n

[2 marks]

b) Next, determine the following (in their appropriate units):
   (i) Damping coefficient c
   (ii) Critical damping coefficient c_{cr}
   (iii) Damping ratio ζ
   (iv) Damped frequency ω_d

[2 marks]

c) Determine the function for the excitation force $F(t)$.

[1 mark]

d) Using convolution integral, determine the response equation $x(t)$.
   (*Hint: At least one pair of exponential terms should cancel out each other.*)

[3 marks]

e) Confirm that the initial displacement x_0 and velocity $\dot{x}_0$ are both zero.
   (*Note: for convolution integral to be applicable, both initial conditions must be zero.*)

[3 marks]

f) The excitation force $F(t)$ in this scenario would diminish in magnitude over time. If there is no damping, predict whether the response of the system would diminish over time or not.

[2 marks]

Continued...

Part 2

Figure Q2_2(a) shows an illustration for the experimental setup to test a design for a linear actuator. Figures (b) and (c) show the possible graphs for the records of the displacement of the spring from its original position.

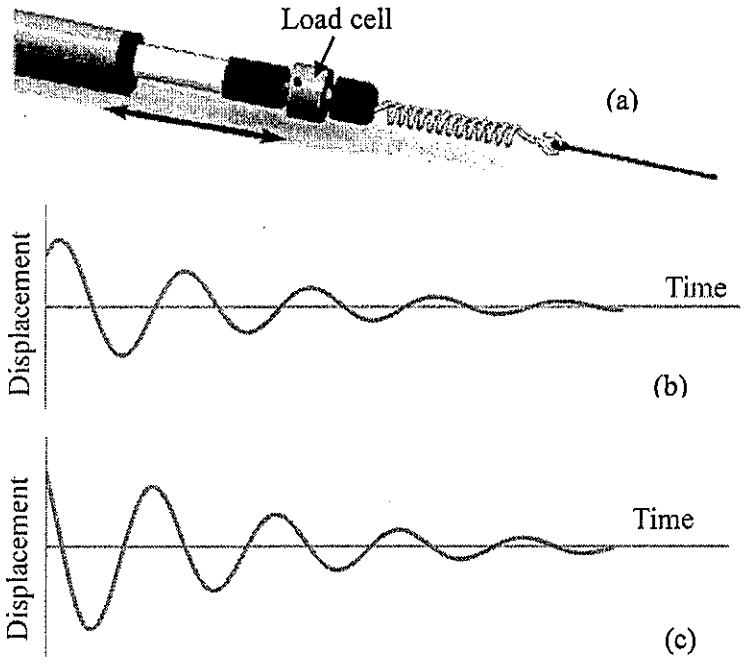

Figure Q2_2

The actuator is to be initially extended, i.e. having a positive amplitude for its displacement at time $t = 0$. There is no source of excitation.

In either graph, the amplitudes of the peaks diminish by half in terms of magnitude across each cycle.

The time period T between each cycle has been measured as approximately 0.22 s. The mass of the design must be 0.432 kg.

a) The results are expected to follow graph (c). Provide a brief explanation for this.
[2 marks]

Continued...

b) The time period of each cycle is too fast for conventional distance measurement. Briefly explain how the displacement can be measured.
(*Hint: Examine Figure Q2_2(a) again.*)

[2 marks]

c) Determine the logarithm decrement δ for this design.

[2 marks]

d) Determine the damping ratio ζ of the design.

[2 marks]

e) Determine the natural frequency ω_n of the design.

[1 mark]

f) Determine the equivalent stiffness k of the design.

[1 mark]

g) Determine the critical damping coefficient c_{cr}.

[1 mark]

h) Determine the actual damping coefficient c.

[1 mark]

Continued...

Question 4 (25 Marks)

There is a damped multi-degree system in horizontal translation, as shown in Figure Q4 (a). Figures Q4(b) and (c) show the free-body diagrams for m_1 and m_2. The effect of gravity is not significant in this case.

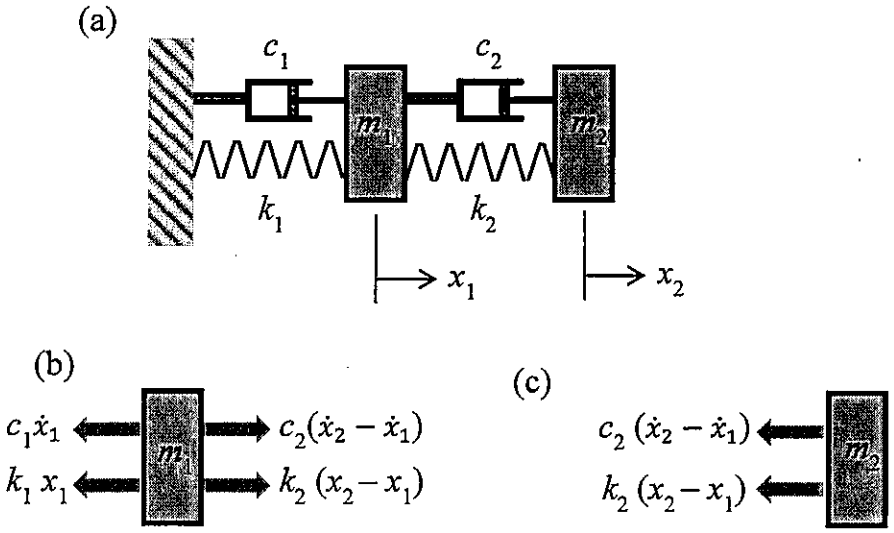

Figure Q4

The quantities in the system are as follows:

$m_1 = 4$ kg $\quad m_2 = 4$ kg $\quad c_1 = 4$ kg · rad/s $\quad c_2 = 4$ kg · rad/s

$k_1 = 30$ N/m $\quad k_2 = 50$ N/m

a) Using Figure Q4(a) and (b), determine the following:
   (i) The kinetic energy of the system, *KE* in terms of $\dot{x}_1$ and $\dot{x}_2$
   (ii) The potential energy of the system, *PE* in terms of x_1 and x_2
   (iii)The dissipation of energy in the system, *DE* in terms of $\dot{x}_1$ and $\dot{x}_2$
   (*Note: include the values for the quantities that have been listed.*)

[3 marks]

b) Apply Lagrange's Equation for the following quantities:
   (i) x_1 and $\dot{x}_1$

[5 marks]

   (ii) x_2 and $\dot{x}_2$

[5 marks]

c) Using the answers from part b), obtain the equations of motion in matrix form.

[2 marks]

d) Determine the square root of the matrix for mass, $[M]^{-1/2}$.

[1 mark]

Continued...

e) Determine the matrix product $[K][M]^{-1/2}$.

[1 mark]

f) Determine the mass-normalized torsional stiffness matrix $\widetilde{K}$.

[1 mark]

g) Calculate the determinant for $\left(\widetilde{K} - \lambda I\right)$ in terms of λ.

[3 marks]

h) Determine the eigenvalues λ_1 and λ_2 for $\widetilde{K}$.

[2 marks]

i) Determine the natural frequencies ω_{n1} and ω_{n2} of the system.

[2 marks]

Continued...

Appendix

Spring constant for an axially loaded column	$k = \frac{AE}{L}$
Natural frequency of a one-degree system in translation with constant stiffness	$\omega_n = \sqrt{\frac{k}{m}}$
Typical appearance of a homogeneous differential equation	$\ddot{x} + f\dot{x} + gx = 0$ Where f and g can be constants or functions of x, or both.
Response of a free undamped oscillating one-degree system in translation and constant natural frequency	$x = x_0\, cos(\omega_n t) + \frac{\dot{x}_0}{\omega_n} sin(\omega_n t)$
Critical damping coefficient	$c_{cr} = 2m\omega_n$
Damping ratio	$\zeta = \frac{c}{c_{cr}}$
Response of a free critically damped one-degree system in translation and constant natural frequency	$x = (A_1 + A_2 t)e^{-\omega_n t}$ $A_1 = x_0 \qquad A_2 = \dot{x}_0 + x_0\omega_n$
Gravitational acceleration	$g = 9.81$ m/s^2
Response of a freely vibrating and Coulomb damped one-degree system in horizontal translation and constant natural frequency	$\dot{x} < 0$: $x = A_1 \cos(\omega_n t) + B_1 \sin(\omega_n t) + \frac{\mu mg}{k}$ $\dot{x} = -\omega_n A_1 \sin(\omega_n t) + \omega_n B_1 \cos(\omega_n t)$ $\dot{x} > 0$: $x = A_2 \cos(\omega_n t) + B_2 \sin(\omega_n t) - \frac{\mu mg}{k}$ $\dot{x} = -\omega_n A_2 \sin(\omega_n t) + \omega_n B_2 \cos(\omega_n t)$
Frequency of free underdamped one-degree system in translation and constant natural frequency	$\omega_d = \omega_n\sqrt{1-\zeta^2}$
Convolution integral for an underdamped system with arbitrary excitation	$x(t) = \frac{e^{-\zeta\omega_n t}}{m\omega_d}\int_0^t \big(F(\tau)\big)e^{\zeta\omega_n\tau}\sin\big(\omega_d(t-\tau)\big)\,d\tau$

Continued...

Appendix (continued)

Logarithm decrement method

$$\delta = \frac{1}{n} ln \frac{X(t)}{X(t+nT)}$$

$$\delta = 2\pi \frac{\zeta}{\sqrt{1-\zeta^2}}$$

$$\delta = \zeta \omega_n T$$

Translational kinetic energy

$$KE = \sum_{i=1}^{n} \frac{1}{2} m_i \dot{x}_i^{\,2}$$

Linear potential energy

$$PE = \sum_{i=1}^{n} \frac{1}{2} k_i x_i^{\,2}$$

Dissipation of energy in translation

$$DE = \sum_{i=1}^{n} \frac{1}{2} c_i \dot{x}_i^{\,2}$$

Lagrange's Equation for freely vibrating damped systems in translation

$$\frac{d}{dt}\left(\frac{\partial(KE)}{\partial \dot{x}_i}\right) - \frac{\partial(KE)}{\partial x_i} + \frac{\partial(DE)}{\partial \dot{x}_i} + \frac{\partial(PE)}{\partial x_i} = 0$$

x_i being the relevant displacement

Mass-normalized stiffness matrix

$$[\tilde{K}] = [M]^{-1/2}[K][M]^{-1/2}$$

2-by-2 identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Determinant of a matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = ad - bc$$

Matrix equation to obtain eigenvalues

$$det(\tilde{K} - \lambda_n I) = 0$$

n being the number of possible eigenvalues.

Quadratic solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

End of Paper